

Progression in Calculation



Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Magnitude of Calculations

Year 1 – $U + U$, $U + TU$ (numbers up to 20) including adding zero, $U - U$, $TU - U$ (numbers up to 20) including subtracting zero, $U \times U$, $U \div U$

Year 2 - $TU + U$, $TU +$ multiples of 10, $TU + TU$, $U + U + U$, $TU - U$, $TU -$ tens, $TU - TU$, $TU \times U$, $U \div U$

Year 3 – add numbers with up to three-digits, $HTU +$ multiples of 10, $HTU +$ multiples of 100, subtract numbers up to three-digits, $HTU - U$, $HTU -$ multiples of 10, $HTU -$ multiples of 100, $HTU - HTU$, $TU \times U$, $TU \div U$

Year 4 - add and subtract numbers with up to four-digits, $ThHTU + ThHTU$, $ThHTU - ThHTU$, add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together, $TU \times U$, $HTU \times U$, $TU \times U$, multiply by zero and one, $TU \div U$, $HTU \div U$

Year 5 – add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, $ThHTU \times U$, $ThHTU \times TU$, $HTU \times TU$, multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by U (including remainders as fractions and decimals and rounding according to the context)

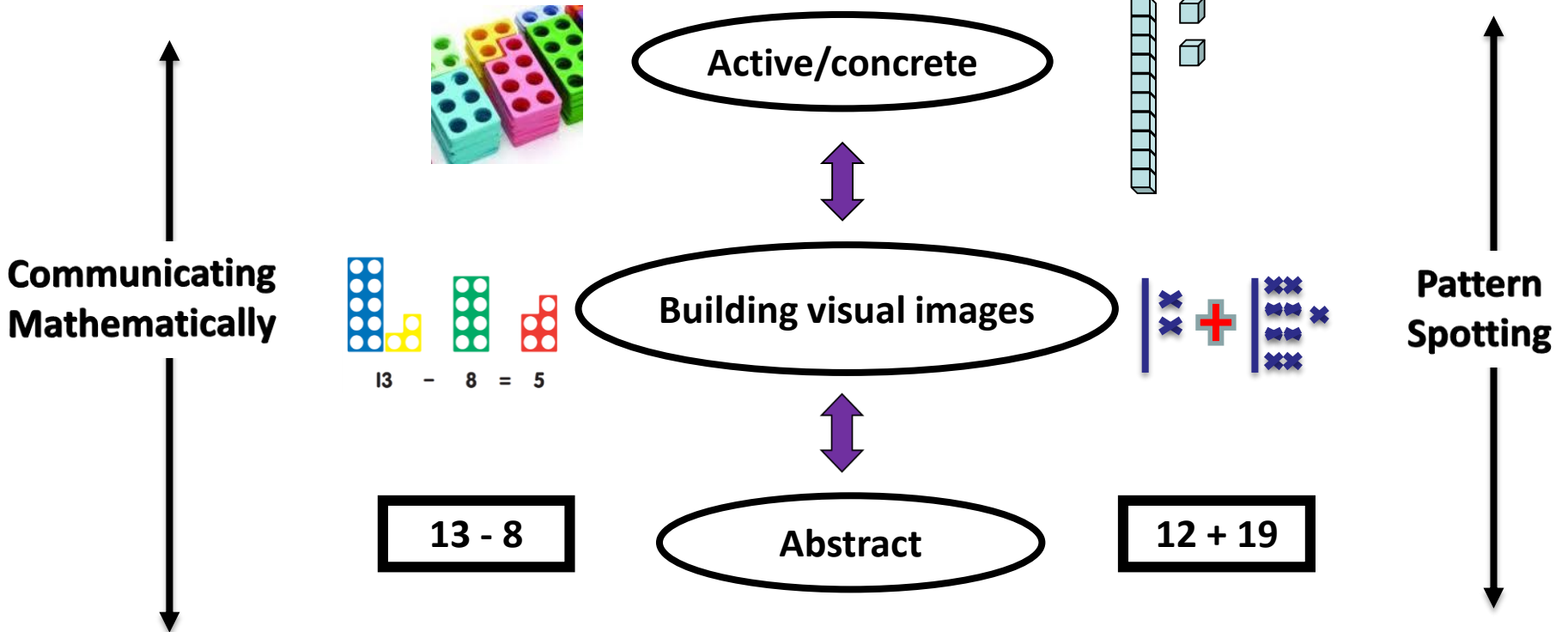
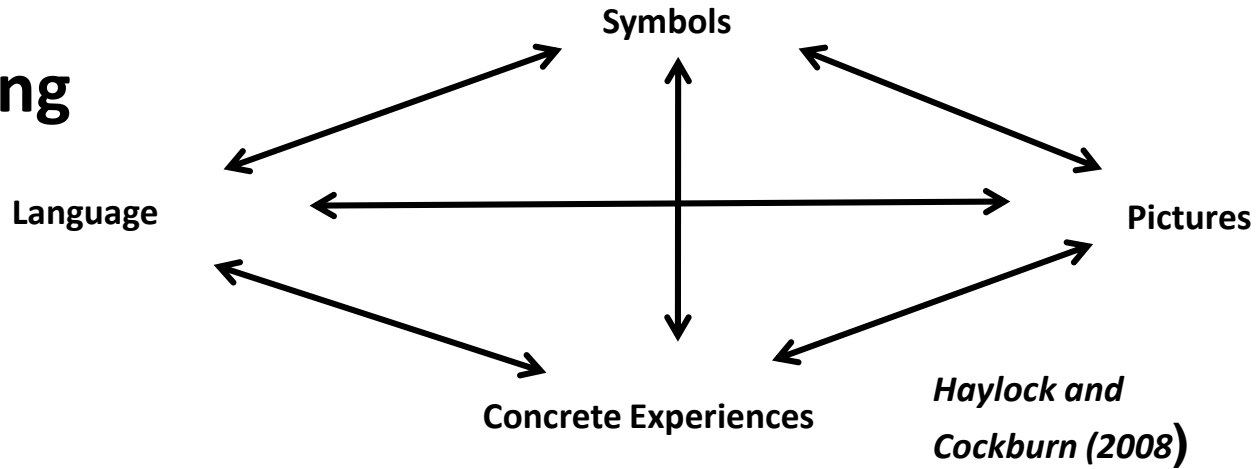
Year 6 - add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by TU , multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by TU (interpreting remainder according to the context), divide decimals up to two-decimal places by U or TU

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. ... pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

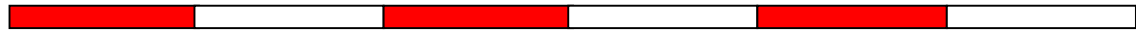
National Curriculum 2014

Structuring Learning

Children must have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.



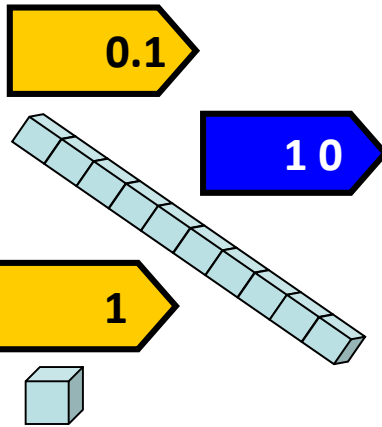
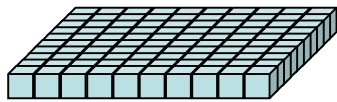
bead string



count stick

place value apparatus

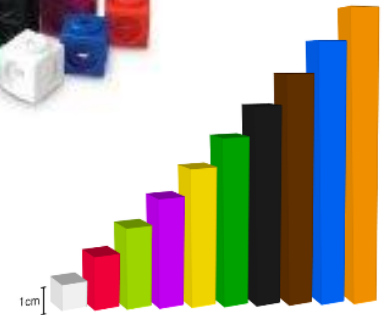
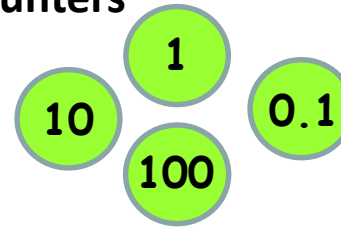
Hundreds	Tens	Units/Ones
100s	10s	1s



Multilink

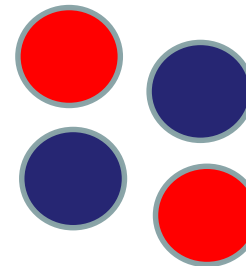
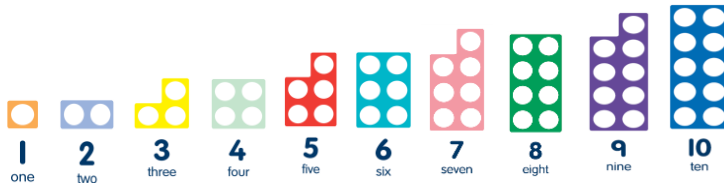


place value counters



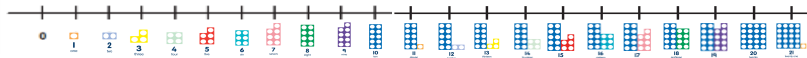
Cuisenaire

Numicon



double sided counters

number line



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
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61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

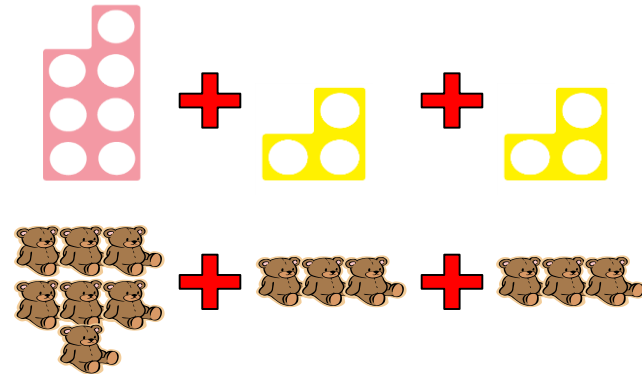
number grids
100 and 200

Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

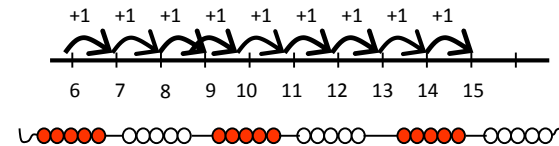
Aggregation

*Union of two sets
How many/much altogether?
The total*



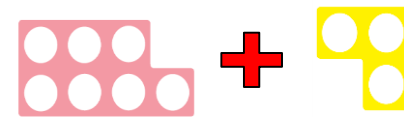
Augmentation

*Start at and count on
Increase by
Go up by*

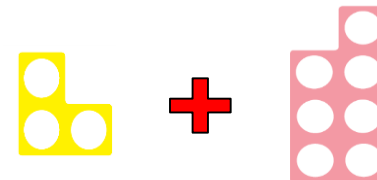


Commutative law

*Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)*



is the same as/equal to (=)



Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Possible Concrete and Visual Representations

Children's Recording

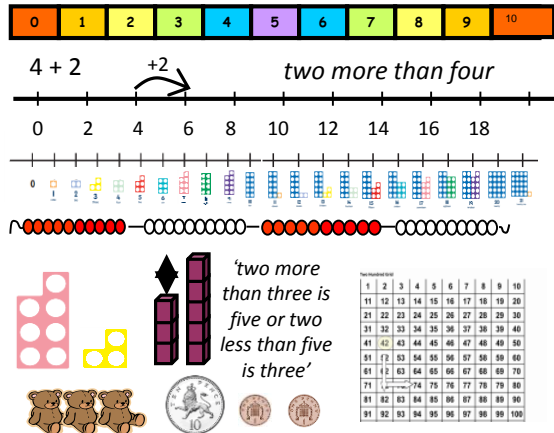
Fluency

Year 1

Children must experience combining two, **and then more than two**, groups of objects using counting on and the language of addition e.g. add, plus

Children must experience increasing numbers e.g. what is two more than seven ?

Compare quantities to say how many less and/or how many more



Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones

If using Numicon, children could use printed Numicon icons and stick these in - progressing to recording number sentences alongside

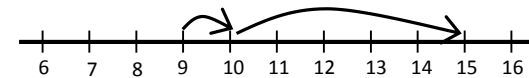
$$1 + 2 = 3$$

Children may record pictorially progressing to recording number sentences alongside

Example

9 and 6
9 + 6

$$9 + 6$$



Count forwards, to and across 100, beginning with 0 or 1 or from any given number

Switch count between tens and ones e.g. 10, 20, 30, 31, 32, 33 ...

Represent and use number bonds up to 20 (establish addition and subtraction as related operations)

Find one more than a number

Find ten more than a number

Count in multiples of 2s, 5s and 10s starting on multiples to highlight pattern recognition

Year 2

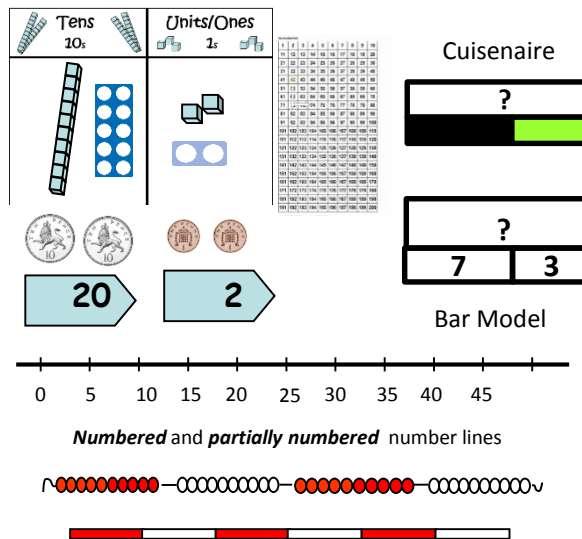
Children should be able to partition numbers in different ways e.g. as 2+2+2+1 or 5+3 or 23 as 20 + 3 or 10+13

Children should use concrete objects, pictorial representations and add numbers in different contexts e.g. money, measures

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE** THAN TWO NUMBERS

Children should understand the language of sum

Ensure children understand that addition is commutative (can be done in any order)



Use Numicon, number grids, place value apparatus/Dienes, place value grids, place value cards, Encourage children to partition numbers rather than counting in ones.

Children apply, develop and secure their understanding of place value

Use jottings and record number sentences

Tens 10s	Units/Ones 1s	
		41
		28
40	1	
+ 20	+ 8	
= 60	= 9	60 + 9 = 69

Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20

Use knowledge to derive and use number facts up to 100

Add numbers mentally including TU + U, TU + tens, TU + TU, U + U + U

End of Year Expectations

Year 3

Add numbers with up to three-digits
(leading to formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** WITH DIFFERING NUMBERS OF DIGITS

Children should partition numbers, up to 1000, in different ways
e.g. $100 + 40 + 6$ or $100 + 30 + 16$

Solve problems in different contexts including missing number problems

Year 4

Add numbers with up to four-digits (formal written column method) including numbers with up to two decimal places in the context of money

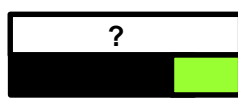
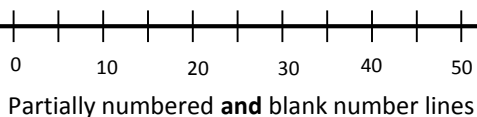
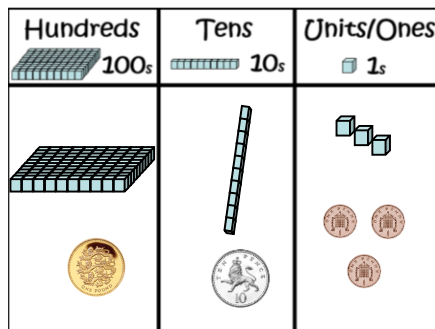
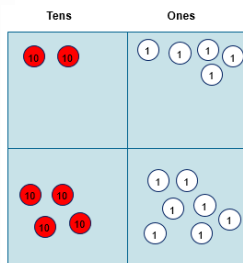
ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve two-step problems in different contexts including missing number problems

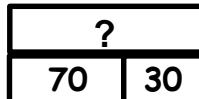
Possible Concrete and Visual Representations

Two Hundred Grid

1	2	3	4	5	6	7	8	9	10
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Cuisenaire



Bar Model

Teacher Modelling/Children's Recording

Children apply, develop and secure their understanding of place value and begin to record in columns

Manipulatives SHOULD be used alongside algorithms

Column addition (no exchanging) with up to three-digits

$$\begin{array}{r} 40 + 1 \\ + 20 + 8 \\ \hline 60 + 9 = 69 \end{array} \qquad \begin{array}{r} 40 + 3 \\ 20 + 8 \\ \hline 70 + 1 = 71 \\ 10 \end{array}$$

Expanded recording without exchange

$$\begin{array}{r} 100 + 40 + 1 \\ + 100 + 20 + 8 \\ \hline 200 + 60 + 9 = 269 \end{array}$$

Expanded recording

Expanded recording with exchange

$$\begin{array}{r} \text{HTU} \\ 141 \\ + 128 \\ \hline 269 \end{array}$$

Compact (column) recording

$$\begin{array}{r} 143 \\ + 128 \\ \hline 271 \\ 1 \end{array} \qquad \text{Column addition (with exchanging)}$$

$$\begin{array}{r} \text{HTU} \\ 789 \\ + 642 \\ \hline 1431 \\ 11 \end{array} \qquad \begin{array}{r} \pounds 7.89 \\ + \pounds 6.42 \\ \hline \pounds 14.31 \\ 11 \end{array}$$

Compact (column) recording

Add decimals in the context of money

Fluency

Count in ones, tens and hundreds maintaining fluency through varied and frequent practice

Count from 0 in multiples of 4, 8, 50 and 100

Find 10 or 100 more than a number

Mentally add HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers, the answer could exceed 100

Count in 6s, 7s, 9s, 25s and 100s

Find 1000 more than a number

Perform mental calculations with increasingly large numbers to aid fluency

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Year 5

Add numbers with more than four-digits and decimals up to three places

(formal written column method)

N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems selecting and justifying methods

Perform mental calculations with increasingly large numbers

Year 6

Add numbers with more than four-digits and decimals up to three places

(formal written column method)

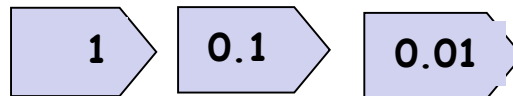
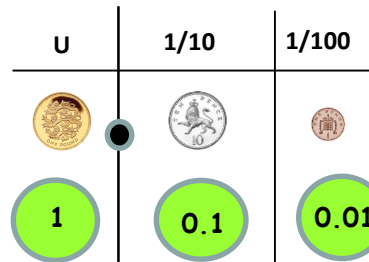
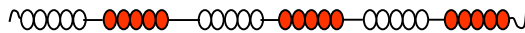
N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS**, INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve more complex calculations mentally

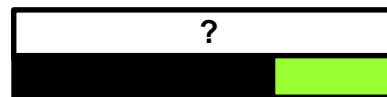
Solve multi-step problems in contexts, deciding which operations and methods to use and why

Possible Concrete and Visual Representations

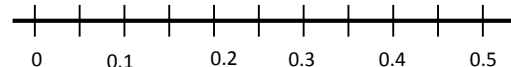
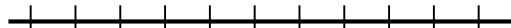
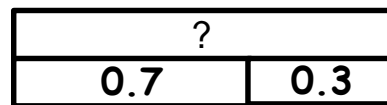
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9



Cuisenaire



Bar Model



Partially numbered and blank number lines



Teacher Modelling/Children's Recording

Manipulatives could be used alongside algorithms

$$\begin{array}{r} 2141 \\ + 1128 \\ \hline 3269 \end{array}$$

Column addition (no exchanging)

$$\begin{array}{r} 21.41 \\ + 1.12 \\ \hline 22.88 \end{array}$$

$$\begin{array}{r} 5189 \\ + 3128 \\ \hline 8317 \\ 11 \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} 51.89 \\ + 3.128 \\ \hline 55.018 \\ 11 \end{array}$$

Addition with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency

Count forwards in powers of ten up to 100000

Count forwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Practise fluency of written methods

Count in tens and hundreds increasing fluency of order and place value

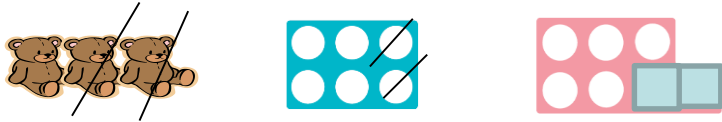
Perform increasingly complex mental calculations and those with increasingly large numbers to aid fluency

Structures of Subtraction (Haylock and Cockburn 2008)

Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

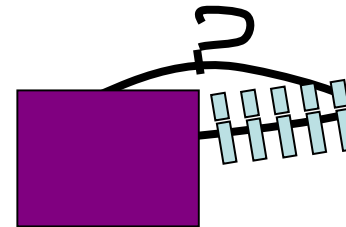
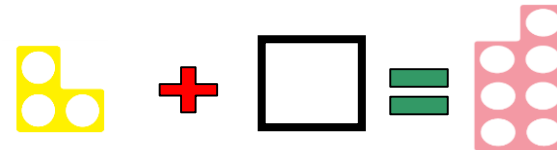
Partitioning

Take away
... how many left?
How many are not?
How many do not?



Inverse-of-addition

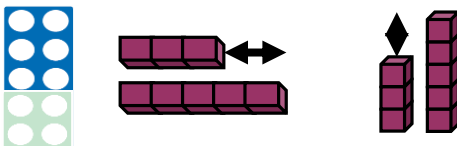
What must be added?
How many (much) more needed?



There are ten pegs
on the hanger –
how many are covered?

Comparison

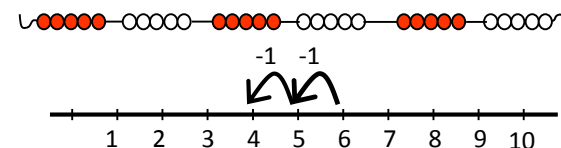
What is the difference?
How many more?
How many less (fewer)?
How much greater?
How much smaller?



'two more than three
is five or two less than
five is three'

Reduction

Start at and reduce by
Count back by
Go down by



Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Possible Concrete and Visual Representations

Teacher Modelling/ Children's Recording

Fluency

Year 3

Subtract numbers with up to three-digits
(formal written column method)
Children apply, develop and secure their understanding of place value and begin to record in columns

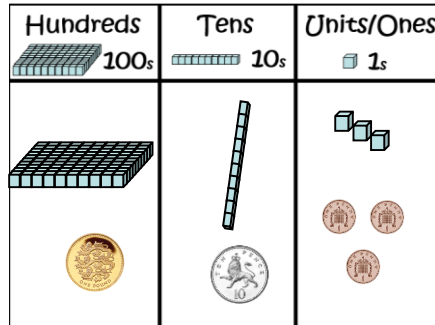
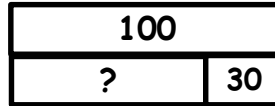
Year Number Grid

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Cuisenaire

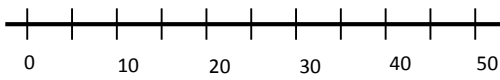
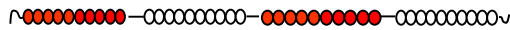
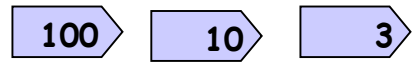
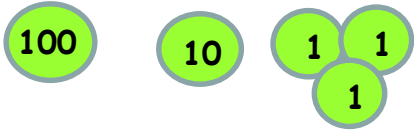


Bar Model



Year 4

Subtract numbers with up to four-digits
(formal written column method)
Understand subtraction as the inverse of addition
Solve two-step problems deciding upon the appropriate operations and methods and justifying choices made



Children **SHOULD** use manipulatives alongside algorithms to transition between practical and abstract

no exchange

$$\begin{array}{r} 68 - 23 \\ \hline \end{array}$$

with exchange

$$\begin{array}{r} 63 - 28 \\ \hline \end{array}$$

$$\begin{array}{r} 60 \quad 8 \\ - 20 \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 50 \quad 60 \quad 10 + 3 \\ - 20 \quad 8 \\ \hline \end{array}$$

$$40 + 5 = 45$$

$$30 + 5 = 35$$

$$148 - 121$$

$$\begin{array}{r} 100 \quad 40 \quad 8 \\ - 100 \quad 20 \quad 1 \\ \hline \end{array}$$

Column subtraction (no exchange)

$$\begin{array}{r} 148 \\ - 121 \\ \hline 27 \end{array}$$

$$0 + 20 + 7 = 27$$

Column subtraction (with exchange)

$$723 - 317$$

$$\begin{array}{r} 1 \quad 1 \\ 7 \quad 2 \quad 3 \\ - 3 \quad 1 \quad 7 \\ \hline 4 \quad 0 \quad 6 \end{array}$$

$$723 - 367$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ 7 \quad 2 \quad 3 \\ - 3 \quad 6 \quad 7 \\ \hline 3 \quad 5 \quad 6 \end{array}$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ \pounds 7 \quad 2 \quad 3 \\ - \pounds 3 \quad 6 \quad 7 \\ \hline \pounds 3 \quad 5 \quad 6 \end{array}$$

Count back in ones, tens and hundreds maintaining fluency through varied and frequent practice
Switch count between hundreds, tens and ones e.g 500, 400, 300, 290, 280, 270, 269, 268, 267
Mentally subtract HTU + ones, HTU + tens, HTU + hundreds
Perform mental calculations with two-digit numbers
Find ten and a hundred less than a number with up to three-digits

Count back in 6, 7, 9, 25 and 1000
Count back through zero to include negative numbers
Find 1000 less than a number
Continue to practise mental calculations with increasingly large numbers to aid fluency

Ensure children can solve calculations where zero is a place holder

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Possible Concrete and Visual Representations

Teacher Modelling/ Children's Recording

Fluency

Year 5

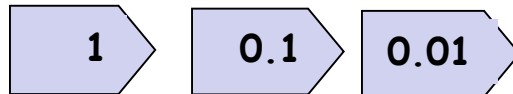
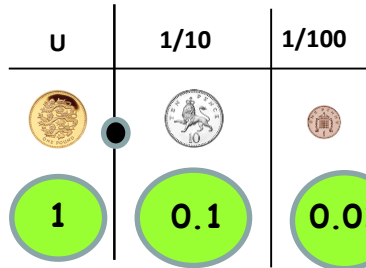
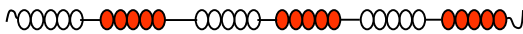
Subtract larger numbers
(formal written column method)

N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS WITH DIFFERING NUMBERS OF DIGITS

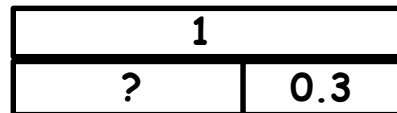
Solve multi-step problems selecting and justifying methods

Subtract numbers mentally with increasingly large numbers

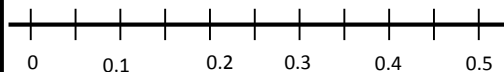
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9



Cuisenaire



Bar Model



Children might use manipulatives alongside algorithms

Column subtraction (no exchanging)

$$\begin{array}{r} 13548 \\ - 12128 \\ \hline 1420 \end{array}$$

Column subtraction
(with exchanging)

$$\begin{array}{r} \overset{2}{1} \overset{13}{3} \overset{11}{4} \overset{1}{2} 3 \\ - 12678 \\ \hline 745 \end{array}$$

Ensure children can solve calculations where zero is a place holder

$$\begin{array}{r} 1.48 \\ - 1.21 \\ \hline 0.27 \end{array}$$

Column subtraction
(no exchanging)

Column subtraction
(with exchanging)

$$\begin{array}{r} \overset{6}{7} \overset{11}{.2} \overset{1}{3} \\ - 3.67 \\ \hline 3.56 \end{array}$$

Subtraction with decimals up to three decimal places including in different contexts e.g. money and measures

Count backwards in powers of ten up to one million

Count backwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Year 6

Subtract multi-digit numbers including numbers with up to three decimal places

(formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems in contexts, deciding which operations and methods to use and why

Solve more complex calculations mentally

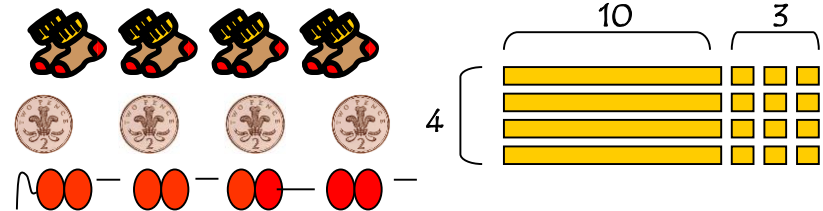
Undertake mental calculations with increasingly large numbers and more complex calculations

Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

Repeated addition

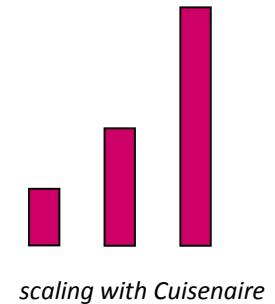
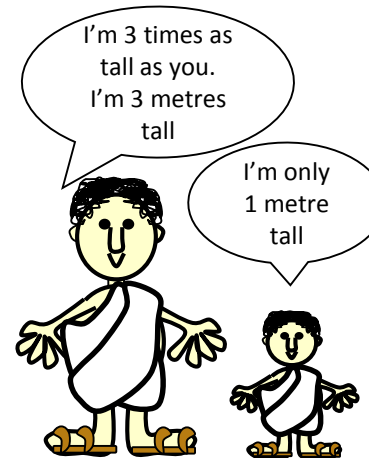
So many lots (sets) of so many
How many (how much) altogether
Per, each



Scaling

Scaling, scale factor
Doubling, trebling

So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

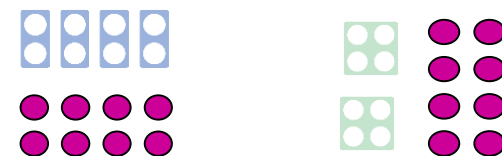


Commutative law

Scaling, scale factor
Doubling, trebling

So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

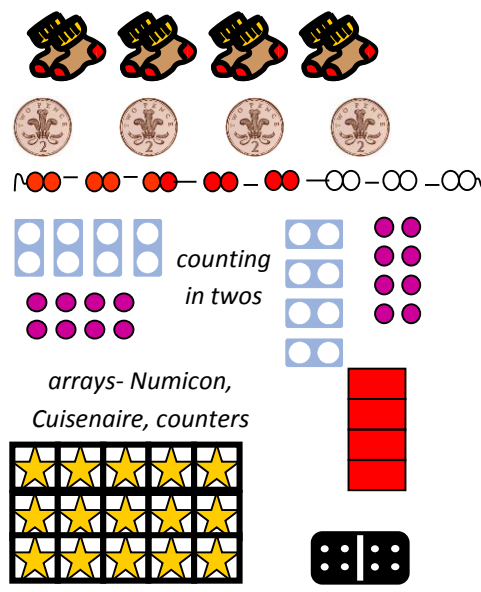
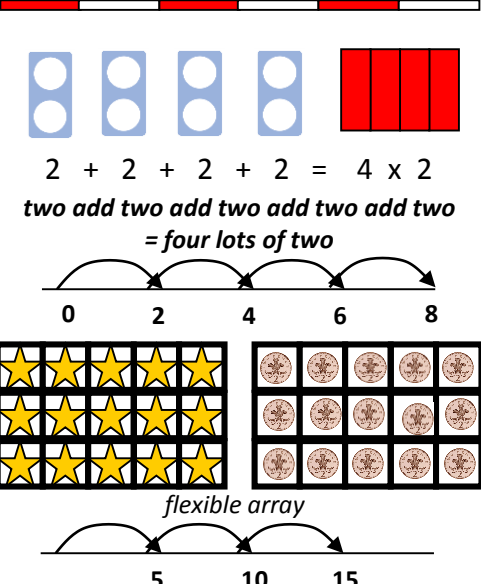
a x b and b x a are equal



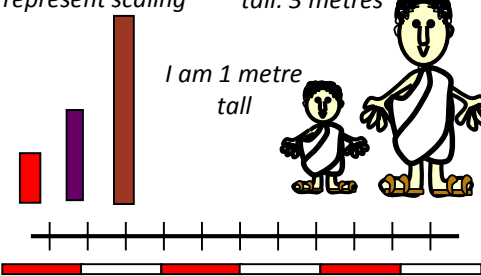
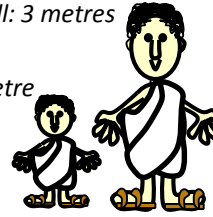


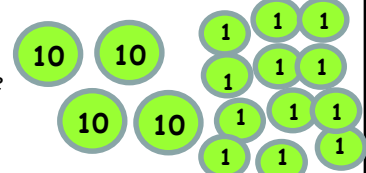
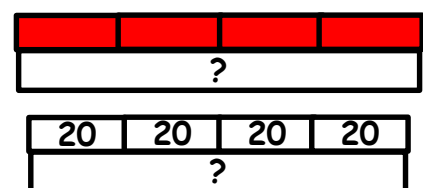
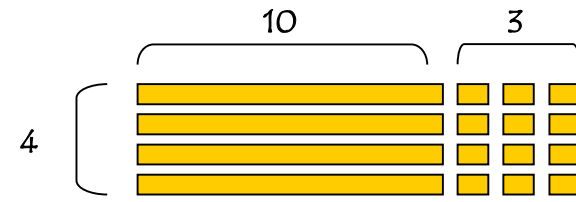
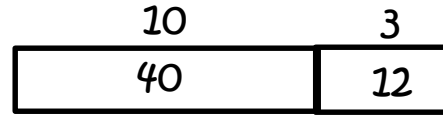

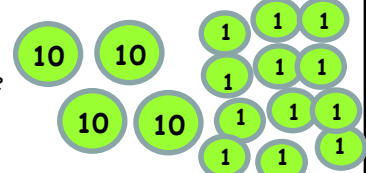
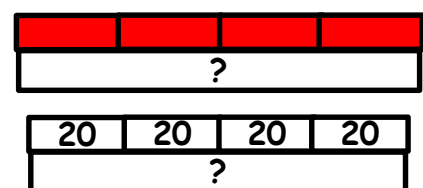
4 x 2 is the same as/equal to 2 x 4

Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly.
 Multiplication and division should be taught together.

End of Year Expectations	Possible concrete and visual representation	Children's Recording	Fluency
<div data-bbox="19 199 193 271" style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">Year 1</div> <p data-bbox="28 328 425 392">Solve single step practical problems involving multiplication</p> <p data-bbox="28 449 425 506">Use concrete objects, pictorial representations to explore grouping</p> <p data-bbox="28 556 425 642">Make connections between arrays, number patterns and counting in twos, fives and tens</p> <p data-bbox="28 721 425 749">Double numbers and quantities</p>	 <p data-bbox="656 442 753 499"><i>counting in twos</i></p> <p data-bbox="502 564 734 621"><i>arrays- Numicon, Cuisenaire, counters</i></p> <p data-bbox="521 785 676 813"><i>flexible array</i></p>	<p data-bbox="1043 214 1449 242">Practical only e.g. link to small world</p> <p data-bbox="1004 264 1506 385">Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p> <p data-bbox="1217 421 1352 485"><i>four lots of two is eight</i></p> <p data-bbox="1217 521 1352 585"><i>two lots of four is eight</i></p> <p data-bbox="1120 742 1352 771"><i>track with cuisenaire</i></p>	<p data-bbox="1603 285 1854 371">Count in twos, fives and tens from different multiples</p> <p data-bbox="1622 392 1835 421">e.g. 6, 8, 10, 12 etc</p> <p data-bbox="1622 442 1835 499">Emphasise number patterns</p> <p data-bbox="1622 521 1835 578">Double number and quantities</p>
<div data-bbox="19 835 193 906" style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">Year 2</div> <p data-bbox="28 935 425 992">Understand multiplication as repeated addition</p> <p data-bbox="28 1021 425 1106">Calculate mathematical statements for multiplication within the tables and write them using symbols</p> <p data-bbox="28 1142 425 1199">Understand and solve problems involving arrays</p> <p data-bbox="28 1220 425 1306">Ensure children understand that multiplication is commutative (can be done in any order)</p> <p data-bbox="28 1335 425 1392">Understand that multiplication and division are inverse operations</p>	 <p data-bbox="502 985 927 1013">$2 + 2 + 2 + 2 = 4 \times 2$</p> <p data-bbox="502 1021 927 1078"><i>two add two add two add two add two = four lots of two</i></p> <p data-bbox="637 1320 792 1349"><i>flexible array</i></p>	<p data-bbox="1043 842 1506 871">Record practical work as number sentences</p> <p data-bbox="1352 935 1468 963">$4 \times 2 = 8$</p> <p data-bbox="1352 992 1468 1021">$2 \times 4 = 8$</p>	<p data-bbox="1584 899 1893 985">Count in twos, threes, fives from zero and tens from any number</p> <p data-bbox="1622 1013 1854 1042">e.g. 6, 8, 10, 12 etc</p> <p data-bbox="1584 1063 1893 1092">Emphasise number patterns</p> <p data-bbox="1584 1113 1893 1228">Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10</p> <p data-bbox="1584 1256 1893 1313">Solve multiplication problems mentally</p>

Multiplication – multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency																																				
<p>Year 3</p> <p>Develop reliable written methods</p> <p>Understand and solve scaling problems</p> <p>Solve problems involving multiplication including correspondence</p>	<p><i>Cuisenaire to represent scaling</i></p>  <p><i>Statue is 3 times as tall: 3 metres</i></p>  <p><i>flexible array</i></p>  <p><i>arrays</i></p>    <p><i>bar models</i></p>	<p><i>Children <u>must</u> use manipulatives alongside algorithms</i></p> <p>4×13 'four <u>lots of</u> thirteen'</p>   <p><i>Expanded methods – grid and area</i></p> <table border="1" data-bbox="985 656 1449 842"> <tr> <td></td> <td>10</td> <td>3</td> <td></td> </tr> <tr> <td>4</td> <td>40</td> <td>12</td> <td></td> </tr> <tr> <td>x</td> <td>10</td> <td>3</td> <td></td> </tr> <tr> <td>4</td> <td>40</td> <td>12</td> <td></td> </tr> </table> <p>$40 + 12 = 52$</p>		10	3		4	40	12		x	10	3		4	40	12		<p>Count from 0 in multiples of 4, 8, 50 and 100</p> <p>Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100</p> <p>Practise mental recall of multiplication tables – 3, 4 and 8x times tables</p> <p>Connect the 2, 4 and 8 times tables using doubling</p> <p>Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g. $4 \times 4 \times 12 = 12 \times 4 \times 5 = 12 \times 20$</p>																				
	10	3																																					
4	40	12																																					
x	10	3																																					
4	40	12																																					
<p>Year 4</p> <p>Multiplying three numbers</p> <p>Solve two-step problems</p> <p>Multiplying by 0 and by 1</p> <p>Develop fluency in short multiplication using formal written layout</p> <p>Solve problems involving multiplication including using the distributive law, integer scaling problems and harder correspondence problems</p>	<p>4×13</p>  <p><i>place value counters</i></p>   <p><i>bar models</i></p>	<p><i>Progressing to developing fluency in short multiplication</i></p> <table border="1" data-bbox="927 985 1487 1256"> <tr> <td>1</td> <td>3</td> <td></td> <td>1</td> <td>3</td> <td>3</td> </tr> <tr> <td>x</td> <td>4</td> <td></td> <td>x</td> <td>4</td> <td></td> </tr> <tr> <td colspan="3"><hr/></td> <td colspan="3"><hr/></td> </tr> <tr> <td>5</td> <td>2</td> <td></td> <td>5</td> <td>3</td> <td>2</td> </tr> <tr> <td colspan="3"><hr/></td> <td colspan="3"><hr/></td> </tr> <tr> <td>1</td> <td></td> <td></td> <td>1</td> <td>1</td> <td></td> </tr> </table> <p><i>Start with digits that are below five so children can practise method without encountering difficulty with multiplication tables</i></p>	1	3		1	3	3	x	4		x	4		<hr/>			<hr/>			5	2		5	3	2	<hr/>			<hr/>			1			1	1		<p>Count in multiples of 6, 7, 9, 25 and 1000</p> <p>Recall and use multiplication facts up to 12×12 with increasing fluency</p> <p>Derive multiplication facts with up to three-digits</p> <p>Recognise and use factor pairs and commutativity in mental calculations</p> <p>Use the distributive law</p> <p>Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. $2 \times 6 \times 5 = 10 \times 6$</p>
1	3		1	3	3																																		
x	4		x	4																																			
<hr/>			<hr/>																																				
5	2		5	3	2																																		
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1			1	1																																			

Multiplication - multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations

Year 5

Multiply decimals with up to three decimal places

Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers

Solve problems involving all four operations where larger numbers are used by decomposing them into their factors

Multiply whole numbers and those involving decimals by 10, 100 & 1000

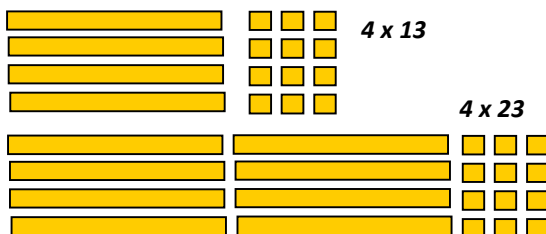
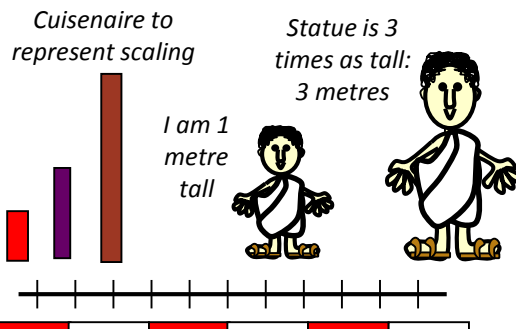
Understand and use multiplication and division as inverses including in problems involving missing numbers and balancing equations

Solve problems involving multiplication and division including scaling by simple fractions

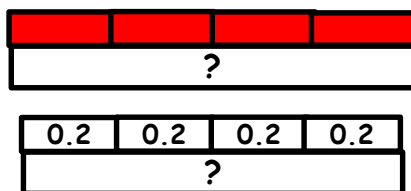
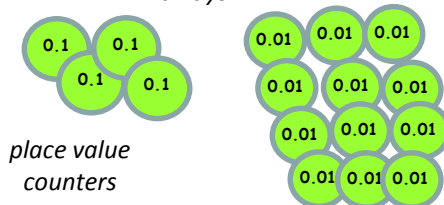
Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)

Recognise and square and cube numbers and associated notation

Possible concrete and visual representation

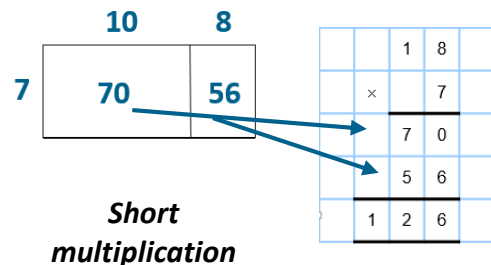


arrays



Teacher Modelling/Children's Recording

Children might use manipulatives alongside algorithms



$$\begin{array}{r} 1324 \\ \times 6 \\ \hline \end{array}$$

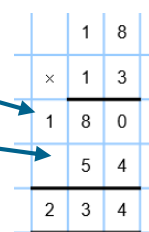
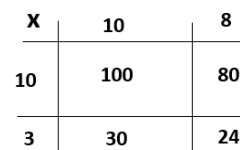
$$\begin{array}{r} 3.24 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7944 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 19.44 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 112 \\ \times 10 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \times 10 \\ \hline \end{array}$$



$$\begin{array}{r} 1324 \\ \times 26 \\ \hline \end{array}$$

$$\begin{array}{r} 3.24 \\ \times 26 \\ \hline \end{array}$$

$$\begin{array}{r} 26480 \\ \times 112 \\ \hline \end{array}$$

$$\begin{array}{r} 64.80 \\ \times 12 \\ \hline \end{array}$$

$$\begin{array}{r} 34424 \\ \times 111 \\ \hline \end{array}$$

$$\begin{array}{r} 84.24 \\ \times 11 \\ \hline \end{array}$$

Fluency

Count forwards in steps of powers of 10 from any given number up to 1 000 000

Practise and extend use of formal written method of short multiplication

Apply all multiplication tables frequently. Commit them to memory and use them confidently to make larger calculations

Multiply numbers mentally drawing upon known facts

Year 6

Multiply numbers up to 4-digit x TU

Multiply numbers with up to two decimal places x whole number

Multiply multi-digit numbers up to four-digits by a two-digit whole number

Multiply single-digit numbers with up to two-decimal places by whole numbers

Solve problems involving all four operations

Undertake mental calculations with increasingly large numbers

Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency

Structures for Division (Haylock and Cockburn 2008)

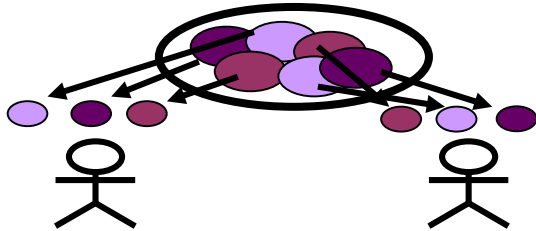
Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

Equal-sharing

Sharing equally between
How many (much) each?

6 shared equally by 2

$$6 \div 2$$

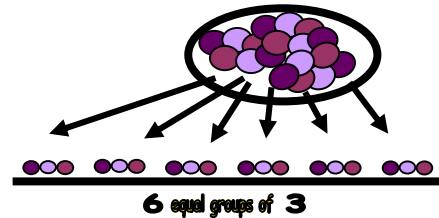


Inverse of multiplication (Grouping)

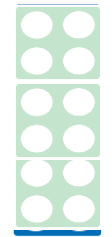
So many lots (sets/groups) of so many
Share equally in to groups of ...

$$18 \div 3$$

18 divided into
equal groups of 3s



Divide twelve into equal
groups of four



Make 12

Overlay
groups of
four

Ratio structure

comparison

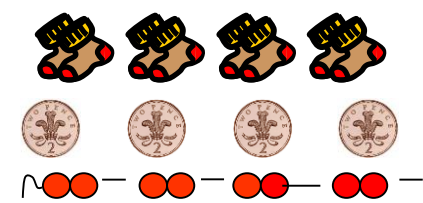

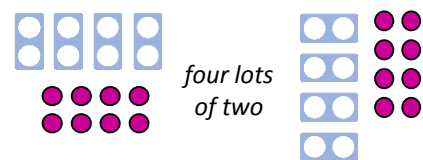
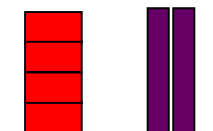
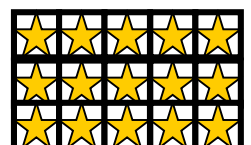

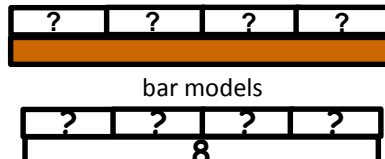
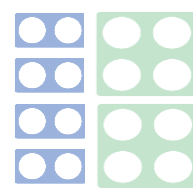
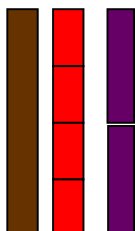

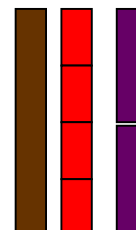
inverse of scaling structure of multiplication
scale factor (decrease)

Barney earns three times more than Fred. If
Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as
long as Ella's. If Jo walks to school in
30 minutes how long does it take Ella?

Division

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly.
Multiplication and division should be taught together.

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<p>Year 1</p> <p>Solve single step practical problems involving division</p> <p>Use concrete objects, pictorial representations</p> <p>Understand division as grouping and sharing</p> <p>Use the language of 'sharing equally between'</p>	<p><i>counting in groups of twos</i></p>  <p><i>straw bundles</i></p>   <p><i>Numicon and counter arrays</i></p>  <p><i>Cuisenaire</i></p> <p><i>four lots of two</i> <i>two lots of four</i></p>  <p><i>doubling</i></p>  <p><i>flexible array</i></p>  <p><i>bar models</i></p>	<p>Practical only e.g. link to small world</p> <p>Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p>  <p><i>Eight can be divided into four equal groups of two or two equal groups of four</i></p> 	<p>Count in twos, fives and tens from different multiples</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise patterns</p> <p>Find simple fractions eg half and quarter, of objects, numbers and quantities</p>
<p>Year 2</p> <p>Solve single step practical problems involving division</p> <p>Use concrete objects, pictorial representations</p> <p>Understand division as grouping</p> <p>Find halves and then quarters</p> <p>Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities and to arrays</p>	<p><i>Record as number sentences using ÷ and =</i></p> <p>$8 \div 4$</p> <p>Eight divided into four equal groups = two in each group</p>  <p>$8 \div 4 = 2$</p>  <p><i>Eight can be divided into four equal groups of two or two equal groups of four</i></p>	<p>Count back in twos, threes, fives from zero and tens from any number</p> <p>e.g. 12, 10, 8, 6 etc</p> <p>Emphasise patterns</p> <p>Connect ten times table to place value and five times table to divisions on a clock face</p> <p>Introduction to multiplication tables. Practise to become fluent in division facts for 2, 5 and 10</p> <p>Solve division problems involving grouping and sharing</p>	

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations

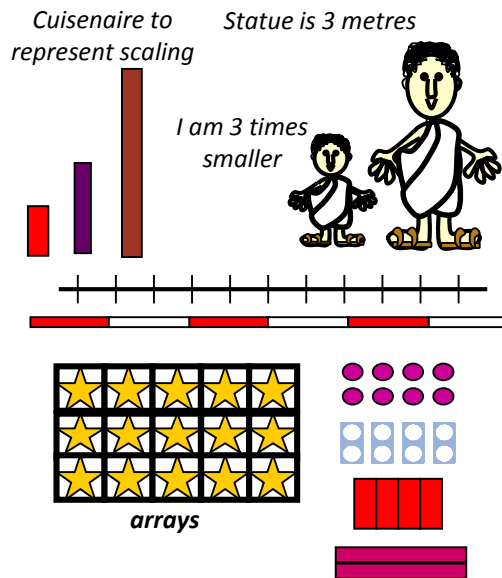
Possible concrete and visual representation

Teacher Modelling/Children's Recording

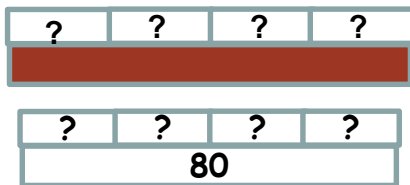
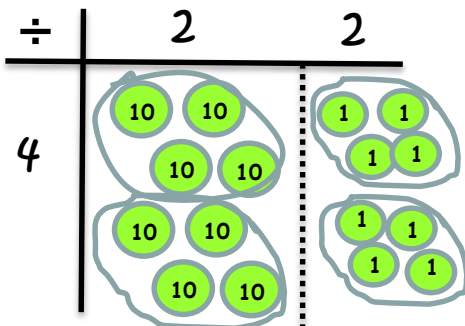
Fluency

Year 3

- Develop a reliable written method for division
- Solve problems involving missing numbers
- Solve problems including those that involve scaling
- Recognise, find and name $\frac{1}{2}$ and $\frac{1}{4}$ of an object, shape or quantity
- Understand the link between unit fractions and division
- Connect $1/10$ to division by 10
- Count in tenths

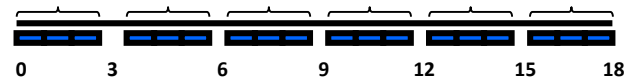


$$88 \div 4$$



bar models

Children should use manipulatives alongside algorithms



Repeated subtraction - chunking

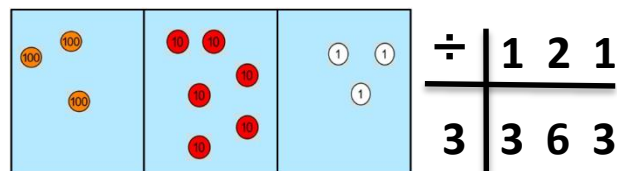
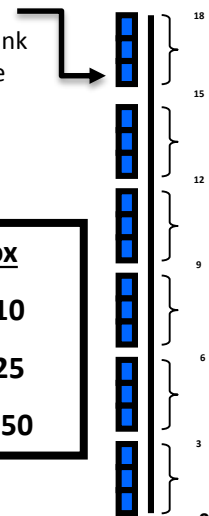
Ensure children see/understand the link between grouping on a number line and vertical recording for chunking

$$95 \div 5 = 19$$

$$\begin{array}{r} 95 \\ - 50 \quad (\underline{10} \times 5) \\ \hline 45 \\ - 25 \quad (\underline{5} \times 5) \\ \hline 20 \\ - 20 \quad (\underline{4} \times 5) \\ \hline 0 \end{array}$$

Fact Box

$$\begin{array}{l} 2 \times 5 = 10 \\ 5 \times 5 = 25 \\ 10 \times 5 = 50 \end{array}$$



$$321 \div 3$$

Short division-
no remainders

$$560 \div 4$$

$$\begin{array}{r} \div \quad 1 \ 4 \ 0 \\ \hline 4 \quad 5 \ 6 \ 0 \end{array}$$

Recall and use related division facts for the 3, 4 and 8x tables (Continue to practise other tables)

Write and calculate mathematical statements for division using what is known

Use division facts to derive related division facts e.g. using $6 \div 3 = 2$ to work out $60 \div 3 = 20$

Continue to practise recalling division facts for multiplication tables up to 12×12

Practise mental methods and extend this to three-digit numbers for example $200 \times 3 = 600$ into $600 \div 3 = 200$

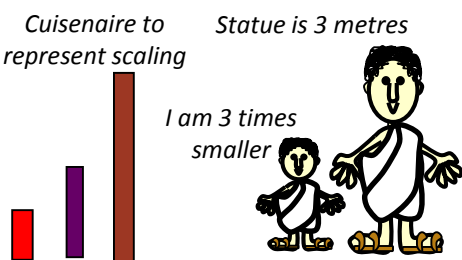

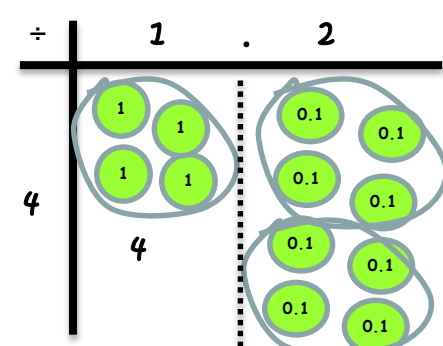
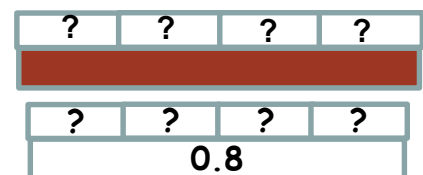
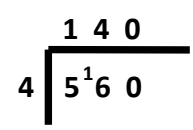
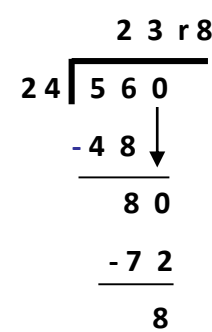
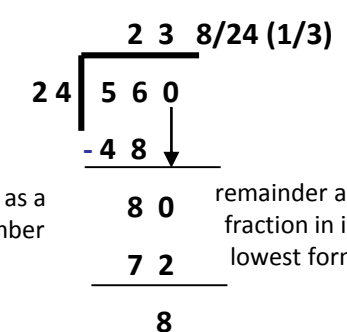
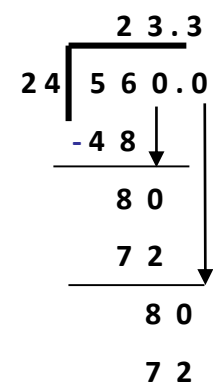

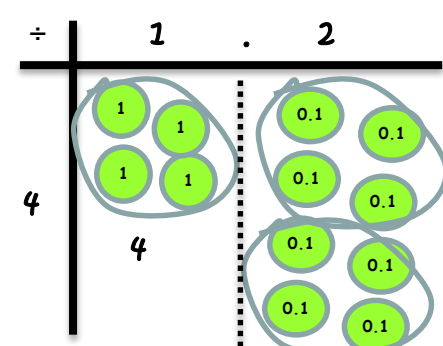
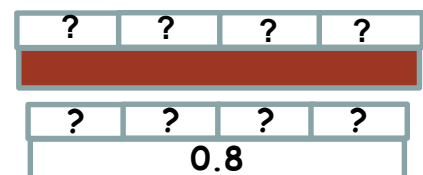
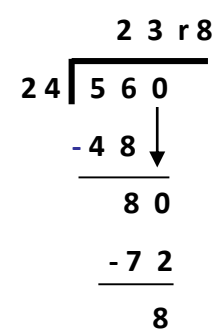
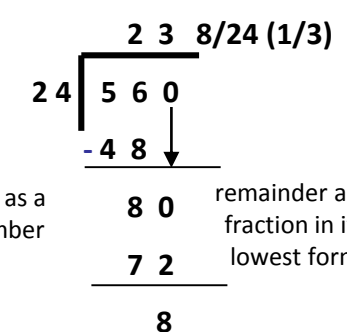
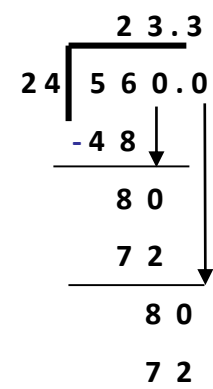
Use place value, known and derived facts to divide mentally, including dividing by 1

Recognise and use factor pairs and commutativity in mental calculations

Year 4

- Become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Divide one- or two-digit numbers by 10 or 100, identifying value of digits as tenths or hundredths
- Solve two-step problems in different contexts, choosing the appropriate operation, working with increasingly harder numbers including correspondence questions e.g. three cakes shared equally between 10 children

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<p>Year 5</p> <p>Identify factors, including finding all factor pairs of a number, and common factors of two numbers</p> <p>Practise and extend the formal written method of short division: numbers up to four-digits by a one-digit number</p> <p>Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding as appropriate for the context</p> <p>Use multiplication and division as inverses</p> <p>Solve problems involving division including scaling down</p> <p>Divide whole numbers and those involving decimals by 10, 100 & 1000</p>	<p><i>Cuisenaire to represent scaling</i></p>  <p><i>flexible arrays</i></p>  <p>$4.8 \div 4$</p>   <p><i>bar models</i></p>	<p><i>Children might use manipulatives alongside algorithms</i></p> <p>without remainder</p> $560 \div 4$  <p>remainder as a fraction</p> $560 \div 24$ <p>long division</p>  <p>remainder as a whole number</p>  <p>remainder as a fraction in its lowest form</p>  <p>remainder as a decimal</p>	<p>Count backwards in steps of powers of 10 for any given number up to 1 000 000</p> <p>Count backwards with positive/negative whole numbers through zero</p> <p>Practise mental calculation with increasingly large numbers</p> <p>Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently to make larger calculations</p>
<p>Year 6</p> <p>Divide numbers up to 4-digits by a 2-digit whole number using formal written methods of long division, interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context</p> <p>Divide numbers with up to 2 decimal places by 1-digit and 2-digit whole numbers, initially in practical contexts involving money and measures</p> <p>Understand the relationship between unit fractions and division</p> <p>Recognise division calculations as the inverse of multiplication</p> <p>Solve problems involving division</p>	<p><i>flexible arrays</i></p>  <p>$4.8 \div 4$</p>   <p><i>bar models</i></p>	<p>long division</p>  <p>remainder as a whole number</p>  <p>remainder as a fraction in its lowest form</p>  <p>remainder as a decimal</p>	<p>Practise division for larger numbers, using the formal written methods of short and long division</p> <p>Continue to use all multiplication tables and division facts to maintain fluency</p> <p>Perform mental calculations, including with mixed operations and larger numbers</p>